

## Erratum on “Mean-field Behavior for the Survival Probability and the Percolation Point-to-Surface Connectivity”

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The purpose of this note is to correct some statement (not the main statement) in the paper.<sup>(2)</sup>

In ref. 2, we considered the critical survival probability (up to time  $t$ ) for oriented percolation and the contact process, and the point-to-surface (of the ball of radius  $t$ ) connectivity for critical percolation. Since the survival probability is a time-oriented version of the point-to-surface connectivity, we denote both quantities by  $\theta_t$  for convenience. The main statement of the paper<sup>(2)</sup> is that, if there is a  $\rho$  such that  $\theta_t/t^{-\rho}$  is bounded away from zero and infinity for all  $t$ , and if both the two-point function and its restricted version exhibit the same mean-field behavior (cf., Assumption 2.1<sup>(2)</sup>), then  $\rho$  also takes on its mean-field value  $\rho_{\text{MF}}$  in high dimensions, i.e.,  $\rho = 1$  for the time-oriented models with  $d > 4$  and  $\rho = 2$  for percolation with  $d > 7$ .

In Section 3.1<sup>(2)</sup>, we proved that the mean-field behavior for the two-point function (i.e., (2.4) and (2.9) in ref. 2 with  $\eta = 0$ ) implies  $\rho \leq \rho_{\text{MF}}$ . This is correct, except that  $d$  has to be greater than 4 for percolation. However, the main statement is not affected by this additional condition since  $d > 7$  for percolation, and remains correct.

The above additional condition is due to applying Proposition 1.7(i) of ref. 1 in order to bound the sum  $\sum_{z \in \mathbb{Z}^d} \|x - z\|^{2-d} \|y - z\|^{2-d}$  in the right-hand side of (3.6) in ref. 2. To apply Proposition 1.7(i) in ref. 1, the sum of the power exponents  $2(2-d)$  has to be less than  $-d$ , and therefore

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$d > 4$ . Heuristically, this can be explained by considering the tail of the convolution  $\sum_{z:|z-x|>2R} \|x-z\|^{2-d} \|y-z\|^{2-d}$ , where  $R = |x-y|$ . By using  $|y-z| \geq \frac{1}{2}|x-z|$ , this tail is bounded by  $O(R^{4-d})$  if  $d > 4$ , and diverges otherwise.

## REFERENCES

1. T. Hara, R. van der Hofstad and G. Slade, Critical two-point functions and the lace expansion for spread-out high-dimensional percolation and related models, *Ann. Probab.* **31**:349–408 (2003).
2. A. Sakai, Mean-field behavior for the survival probability and the percolation point-to-surface connectivity, *J. Statist. Phys.* **117**:111–130 (2004).